SETON HALL UNIVERSITY

TWENTYFIFTH ANNUAL

JOSEPH W. ANDRUSHKIW

MATHEMATICS COMPETITION

1. The sum of the squares of four consecutive positive integers is 1374. Find the smallest of the four consecutive integers.

2. Find the smallest real number, x, for which $\dfrac{2x^2+3x}{3x^2-10x+8} \le 0$.

3. Let L(N) denote the sum of the smallest and the largest prime factors of the integer N, N > 2. For example L(750) = 7 (since $750 = 2 \cdot 3 \cdot 5^3$ and 2 + 5 = 7) and L(81) = 6 (since $81 = 3^4$ and 3 is both the smallest and largest prime factor of 81 so that L(81) = 3 + 3 = 6). Find the smallest four-digit positive integer N for which L(N) = 4.

4. A committee composed of either 4 or 5 members is to be formed; members are to be chosen from 5 people in group A, 3 people in group B or 4 people in group C. At least one from group B must be chosen and at most 3 from any of the three groups may be chosen. How many such committees can be formed?

5. The horizontal line with equation y = -14 is tangent to the graphs (on a coordinate plane) of both $y = 5\cos(2x) - 3\sin(x) + 3c$ and $y = 4\csc(x-\pi) + 5c$; where c is an integer. Find c.

6. Express the number $N = .472397239... = .4\overline{7239}$ in rational form (i.e. in the form $\dfrac{n}{m}$ where n and m are positive integers), reduced to lowest terms.

7. Four integers are randomly chosen from the set S = { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10} without replacement. Find the probability that the four integers chosen (when arranged in order of size) form an arithmetic progression.

8. Consider equations I: $x^2 + bx + c = 0$ and II: $x^2 + dx + f = 0,$ where b, c, d, and f are nonzero real numbers. The discriminant of equation I is 5. The roots of equation I are triple the roots of equation II. Find the discriminant of equation II.

9. Find all ordered triples (x, y, z) of real numbers which simultaneously satisfy the equations:

$$\begin{aligned} x^2 + y^2 - z^2 &= 1 \\ x + y - 5z &= 2 \\ 2x - y + 2z &= 1 \end{aligned}$$

10. Right triangle QP_0P_4 has base QP_0 and right angle QP_0P_4. Side P_0P_4 is divided into four segments by points P_1, P_2, P_3 with P_1 between P_0 and P_2, P_2 between P_1 and P_3, P_3 between P_2 and P_4. $\overline{QP_0}$ is 240 feet long, $\overline{P_0P_4}$ is 180 feet long. Denote the area of triangle QP_0P_1 by A_1, of triangle QP_1P_2 by A_2, of triangle QP_2P_3 by A_3 and of triangle QP_3P_4 by A_4. If A_4 exceeds A_2 by 7680 ft^2, A_3 exceeds A_2 by 1680 ft^2, and A_1 exceeds A_2 by 4560 ft^2, find the area and perimeter of triangle QP_2P_3.

11. Find all positive real numbers x which satisfy the equation $64(\log_{16} x)^4 + 136(\log_{16} x)^3 + 86(\log_{16} x)^2 + 11(\log_{16} x) - 3 = 0$.

12. Consider the complex numbers 1 + i and $-\sqrt{3} - i$, where $i^2 = -1$. Find the largest positive 2-digit integer N for which $(1+i)^N$ is a negative integer and $\left(-\sqrt{3}-i\right)^N$ is a positive integer.

13. The ellipse $16x^2 + 25y^2 = 400$ and the parabola $y^2 = 12x$ lie on a coordinate plane and intersect in two points, A and B. Find an equation of the circle with center at the origin which passes through points A and B.

14. A train goes from A to B to C to D, a distance of 319 miles. The distance from A to B is 33 miles more than the distance from B to C and the distance from B to C is 11 miles more than the distance from C to D. On "Slow Day", the train travelled at rate r_1 from A to B, then at a rate (r_2) half of r_1 from B to C, and then at a rate (r_3) two-thirds of r_1 from C to D. The usual rate for the entire trip is r_1. It took 13/6 hours longer on "Slow Day" from A to D than it usually takes. Find the rate r_1.

15. Let $P_1 = x+1$, $P_2 = (x+1)(x^2-1)$ and $P_3 = (x+1)(x^2-1)(x^3+1)$, where x is a real number and not an integer. Find a rational number x which is a solution of the equation $\dfrac{x}{P_1} - \dfrac{P_1(x-1)}{P_2} - \dfrac{P_2(x^3-1)}{P_3} = \dfrac{-100}{243P_3}$.

16. Triangle ABC has base BC with points D, E and F on BC, D between B and E, E between D and F, and F between E and C. Line segment AD is perpendicular to side BC at D, the length of side AB is 2 inches, and the degree measure of each of the angles BAD, DAE, EAF, FAC is 15°. By how many inches does the perimeter of triangle AFC exceed the perimeter of triangle AEF? (Give the answer in exact form.)